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Supersymmetry and R-symmetry breaking in models with non-canonical Kähler potential

L.G. Aldrovandi* **and D. Marqués***

Departamento de Física-IFLP, Universidad Nacional de La Plata,
C.C. 67, 1900 La Plata, Argentina
E-mail: aldrovandi@fisica.unlp.edu.ar, diego.marques@fisica.unlp.edu.ar

Abstract: We analyze several aspects of R-symmetry and supersymmetry breaking in generalized O'Raifeartaigh models with non-canonical Kähler potential. Some conditions on the Kähler potential are derived in order for the non-supersymmetric vacua to be degenerate. We calculate the Coleman-Weinberg (CW) effective potential for general quiral non-linear sigma models and then study the 1-loop quantum corrections to the pseudo-moduli space. For R-symmetric models, the quadratic dependence of the CW potential with the ultraviolet cutoff scale disappears. We also show that the conditions for R-symmetry breaking are independent of this scale and remain unchanged with respect to those of canonical models. This is, R-symmetry can be broken when generic R-charge assignments to the fields are made, while it remains unbroken when only fields with R-charge 0 and 2 are present. We further show that these models can keep the runaway behavior of their canonical counterparts and also new runaway directions can be induced. Due to the runaway directions, the non-supersymmetric vacua is metastable.

Keywords: Sigma Models, Supersymmetry Breaking, Supersymmetric Effective Theories.

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Contents

1. Introduction

The first proposals for dynamical supersymmetry (SUSY) breaking appeared to be rather non-generic (for a review see [1]), because several classic constrains [2] hardly restricted model building. These constraints are removed if metastability for the vacua is accepted, this giving rise to new possibilities for model building. In fact, by demonstrating the metastable structure of the vacuum in massive $\mathcal{N} = 1$ SQCD (ISS model), it was shown in [3] that metastable dynamical supersymmetry breaking is much more generic and simpler than was previously thought. In the low energy limit of this model (and other SUSY gauge theories), O'Raifeartaigh-type models [4] arise naturally and dynamically, and are therefore appealing candidates for the hidden sector of low-scale supersymmetric theories [5]. Increasing efforts have been made to characterize common aspects of supersymmetry breaking in these models. Among the many common features that are sheared by these generic theories with metastable vacua one should mention

- Supersymmetry breaking and R-symmetry are connected. It was shown in [6] that the existence of an R-symmetry is a necessary condition for supersymmetry breaking, and a spontaneously broken R-symmetry is sufficient. When the theory is not R-symmetric, it can contain supersymmetric vacua.

- Runaway directions are in general present implying that the SUSY-breaking minima is only local [7]. This vacua can be taken to be sufficiently long lived. When the R-symmetry is softly broken supersymmetric vacua can appear, but they can be pushed far away in field space [8, 9].

- The supersymmetry breaking vacua is degenerate at tree-level [10]. It corresponds to a pseudo-moduli space further lifted by quantum corrections. If an exact R-symmetry is present at the classical level, the corrections determine whether R-symmetry is broken or not through the 1-loop effective Coleman-Weinberg (CW) potential [11].

- R-symmetry can be broken at the quantum level when the R-charge assignment to the fields is generic [12]. When fields with only R = 0, 2, are present in the theory, R-symmetry cannot be broken.

- The effective potential includes a quartic divergent term proportional to Λ^4 STr 1, and a quadratic divergent term proportional to Λ^2 STr $\mathcal{M}^2$, with Λ the UV cut-off scale. Both vanish in renormalizable supersymmetric theories.

These items are only shared by theories with canonical Kähler potentials, i.e. those in which the Kähler metric is the identity. For instance, it is not necessarily true that models with non-canonical Kähler posses non-supersymmetric degenerate vacua manifolds. Moreover, the theories are not renormalizable and quadratic and quartic divergent terms appear in the CW effective potential, making them very sensitive to variations of the energy scale. These are (some of) the reasons why generic aspects of supersymmetry breaking in these kind of models have not been deeply studied.

This paper is devoted to study several aspects of O'Raifeartaigh-type models with non-canonical Kähler potentials. Interestingly, not so restrictive conditions have to be imposed on the Kähler metric, in order for the theory to share the above mentioned properties. We start showing some conditions on the Kähler potential that imply degenerate vacua at tree level. We then analyze the properties of this vacua, and show that its characterization is completely analogous to that of the canonical Kähler models. We also show sufficient conditions for these theories to have runaway directions, making the non-supersymmetric minima metastable.

There are many situations in which non-canonical Kähler potentials arise. The theories we consider are conceived as low-energy effective theories of more fundamental renormalizable theories. Loops of modes from these high energy theories induce effective Kähler potentials [3]. It is worth noticing that, since supergravity corrections are neglected, the scales associated to the higher order terms in the Kahler potential are assumed to be much smaller than the Planck scale. Non-minimal Kähler potentials also arise in finite temperature and supergravity theories and were studied in the context of metastability in [9, 13].

The outline of this paper is as follows. In section 2, we derive sufficient conditions on the Kähler, in order for the scalar potential to have a tree-level degenerate non-SUSY vacua. The most general Kähler potential satisfying these conditions for a generalized

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O'Raifeartaigh model is constructed. This derivation is done for a one-dimensional pseudo-moduli space, and we also generalize the result to the case of higher dimensional pseudo-moduli space. In section 3, the model is analyzed in more detail and its main characteristics are discussed. Section 4 is devoted to the analysis of R-symmetry breaking, based on the 1-loop quantum lifting of the flat directions of the pseudo-moduli space. In section 5 we study the non-canonical version of an O'Raifeartaigh model introduced by Shih [12], providing an explicit realization of the main results of this paper. In section 6 we present a summary and a discussion of our results. Finally, we add an appendix with the computation of the 1-loop CW effective potential for a general supersymmetric non-linear sigma model.

2. Degeneracy for non-canonical Kähler

The non-SUSY vacua of renormalizable Wess-Zumino models always consists of a tree-level pseudo-moduli space lifted by quantum corrections. If the theory is R-symmetric, the lifting determines if R-symmetry is broken or not. In general, when studying these theories, one usually relaxes the condition of genericity on the superpotential, in order to obtain a deeper understanding of the SUSY breaking properties of concrete models. These models, although non generic, are quite general (see for example the models of [12]), consisting in families of models sharing similar properties.

When we turn the attention to non-renormalizable theories with non-canonical Kähler potential, there need not be a moduli parameterizing the vacua: non-canonical corrections to the canonical Kähler potential lift the moduli space at tree level. As this lifting depends exclusively on the form of the Kähler potential, it is much more difficult to analyze general aspects of SUSY breaking in non-renormalizable models.

However, we can relax the genericity condition on the Kähler potential (we call "generic" to those Kähler potentials containing all the terms consistent with the symmetries of the theory), and try to look for families of Kähler potentials sharing SUSY breaking and R-symmetry breaking properties.

In this paper, as a first step in analyzing general aspects of SUSY and $\mathrm{U}(1)_R$ breaking in non-renormalizable models, we focus on those families of non-canonical models that share the properties of their canonical counterparts, which were deeply studied and are well understood. Then, as a first step in our analysis, we must look for conditions on the Kähler potential in order for the theory to have a pseudo-moduli space.

One can think of a further step in the study of non-canonical models, as that in which the conditions we derive are relaxed, implying a tree level lifting of the moduli. We shall also make very brief comments about this possibility in this section, but only superficially, leaving this for further research.

2.1 Sufficient conditions for degenerate vacua

This section is devoted to find a set of sufficient conditions on a general Kähler potential K and the superpotential W that imply degeneracy of the (supersymmetry-breaking) vacua.

Let us first review what happens in the case of a theory with canonical Kähler potential $K = \phi^a \delta_{a\bar{a}} \bar{\phi}^{\bar{a}}$ (here $a = 1, \ldots, N_\phi$ label N_ϕ chiral fields ϕ^a). In this case one can show [10]

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that if the potential $V = \bar{W}_{\bar{a}}\delta^{\bar{a}a}W_a$ admits a local non-supersymmetric vacuum, then a set of vacua with the same tree-level energy forming a (continuous) submanifold of the field space necessarily exists. More in detail, from the conditions for a field configuration $\phi_0, \bar{\phi}_0$ to be a non-supersymmetric vacuum:

- $W_a|_{\phi_0} \neq 0$
- $\bar{W}_{\bar{a}}\delta^{\bar{a}a}\partial_a W_b|_{\phi_0,\bar{\phi}_0} = 0$
- $\delta V \geq 0$ at the leading order in the variations $\delta\phi^a, \delta\bar{\phi}^{\bar{a}}$ for any $\delta\phi^a, \delta\bar{\phi}^{\bar{a}}$

one can prove that

$$\bar{W}_{\bar{a}_1}\delta^{\bar{a}_1 a_1}\dots\bar{W}_{\bar{a}_n}\delta^{\bar{a}_n a_n}\partial_{a_1\dots a_n}W_b|_{\phi_0,\bar{\phi}_0} = 0\,, \qquad \forall\, n \geq 1\,. \tag{2.1}$$

Clearly, this result implies that

$$V(\phi_0^a + z\bar{W}_{\bar{a}}\delta^{\bar{a}a}, \bar{\phi}_0^{\bar{a}} + \bar{z}\delta^{\bar{a}a}W_a) = V(\phi_0^a, \bar{\phi}_0^{\bar{a}})\,, \tag{2.2}$$

for any complex z, and then the potential is degenerate at tree-level.

The latter theorem only holds for a canonical Kähler potential. In fact, the vacuum need not to be degenerate for a generic Kähler potential, as can be easily verified through the following simple counter-example presented in [3]. Consider a theory containing a single chiral superfield X, with linear superpotential with coefficient f

$$W = fX\,, \tag{2.3}$$

and an arbitrary Kähler potential $K(X, \bar{X})$. The scalar potential is

$$V = (\partial_X\partial_{\bar{X}}K)^{-1}|f|^2\,. \tag{2.4}$$

Let us suppose that the Kähler potential K is smooth. For smooth K, the potential (2.4) is non-vanishing, and thus there is no supersymmetric vacuum. It is also clear that the vacuum is not necessarily degenerate. Consider, for instance, the behavior of the system near a particular point, say $X \approx 0$. Let

$$K = \bar{X}X - c(\bar{X}X)^2 + \dots \tag{2.5}$$

with positive c. Then there is a locally stable non-supersymmetric vacuum at $X = 0$ and no degeneracy at all.

In spite of this, we will show below that under certain assumptions on the Kähler potential, the presence of a degenerate vacuum can be guaranteed.

First of all, let us note that since we only consider regular (non-smooth K signals the need to include additional degrees of freedom at the singularity) and positive-definite Kähler metrics, the conditions a given vacuum must satisfy to break supersymmetry depend only on the superpotential W and not on the form of the Kähler potential. That is, if the metric $K_{a\bar{a}}$ is regular and positive-definite, the potential $V = \bar{W}_{\bar{a}} K^{\bar{a}a} W_a$ in the vacuum will vanish if and only if the vector W_a is null in that vacuum.[1]

Let us now prove the theorem which guarantees the existence of a tree-level moduli space. We require the following conditions to be satisfied:

- $W_a|_{\phi_0} \neq 0$ (2.6)
- $\bar{W}_{\bar{a}} \partial_b (K^{\bar{a}a} W_a)|_{\phi_0,\bar{\phi}_0} = 0$ (2.7)
- $$\delta V = \sum_{n,p=0}^{\infty} \frac{1}{n!p!} \sum_{m=0}^{n} \sum_{q=0}^{p} \binom{n}{m} \binom{p}{q} \delta\bar{\phi}^{\bar{a}_1} \ldots \delta\bar{\phi}^{\bar{a}_n} \delta\phi^{a_1} \ldots \delta\phi^{a_p}$$ $$\partial_{\bar{a}_1 \ldots \bar{a}_{n-m}} \bar{W}_{\bar{a}}\, \partial_{\bar{a}_{n-m+1} \ldots \bar{a}_n a_1 \ldots a_{p-q}} K^{\bar{a}a}\, \partial_{a_{p-q+1} \ldots a_p} W_a|_{\phi_0,\bar{\phi}_0} \geq 0 \qquad \forall \delta\phi^a, \delta\bar{\phi}^{\bar{a}} \quad (2.8)$$
- $\frac{d}{d\lambda} W_a|_{\phi_0} = 0$ (2.9)
- $\frac{d^m}{d\bar{\lambda}^m} \frac{d^n}{d\lambda^n} K^{\bar{a}a}|_{\phi_0,\bar{\phi}_0} = 0\,, \qquad \forall\, m,n \geq 0 \,/\, m+n > 0\,,$ (2.10)

where $d/d\lambda$ and $d/d\bar{\lambda}$ are defined by

$$\frac{d}{d\lambda} = \bar{W}_{\bar{a}} K^{\bar{a}a}|_{\phi_0,\bar{\phi}_0} \partial_a\,, \qquad \frac{d}{d\bar{\lambda}} = K^{\bar{a}a} W_a|_{\phi_0,\bar{\phi}_0} \partial_{\bar{a}}\ . \tag{2.11}$$

The conditions (2.6)–(2.8) imply that the field configuration $\phi_0, \bar{\phi}_0$ is a non-supersymmetric vacuum of the theory. Concerning conditions (2.9), (2.10), their meaning become clearer by noticing that $d/d\lambda$ and $d/d\bar{\lambda}$ are the derivatives along the curve $\phi^a(\lambda)$, $\bar{\phi}^{\bar{a}}(\bar{\lambda})$ given by

$$\phi^a(\lambda) = \bar{W}_{\bar{a}} K^{\bar{a}a}|_{\phi_0,\bar{\phi}_0} (\lambda - \lambda_0) + \phi_0^a\,, \qquad \bar{\phi}^{\bar{a}}(\bar{\lambda}) = K^{\bar{a}a} W_a|_{\phi_0,\bar{\phi}_0} (\bar{\lambda} - \bar{\lambda}_0) + \bar{\phi}_0^{\bar{a}}\ . \tag{2.12}$$

Therefore, equation (2.10) implies that $K^{\bar{a}a}$ is constant along the curve given by eq. (2.12)

$$K^{\bar{a}a}(\phi(\lambda), \bar{\phi}(\bar{\lambda})) = K^{\bar{a}a}(\phi_0, \bar{\phi}_0)\,, \tag{2.13}$$

with λ any complex number.[2]

Let us prove by recurrence that under the latter assumptions (2.6)–(2.10) the potential V is always degenerate at tree-level. To do this we suppose, as a recurrence condition, that for some non-zero integer n we have

$$\frac{d^k}{d\lambda^k} W_a|_{\phi_0} = 0\,, \qquad \forall\, 1 \leq k \leq n\ . \tag{2.15}$$

[1] An obvious corollary of this is that the connection between R symmetry and supersymmetry breaking pointed out by Nelson and Seiberg [6] is valid for any regular Kähler potential. If the Kähler (and therefore the theory) is not R-symmetric, the N-S argument still holds as long as the superpotential has R-charge $R(W) \neq 0$.

[2] In other words, if we denote $\vec{U}$ to the tangent vector to the curve (2.12) (i.e. the vector field with components $U^a = \bar{W}_{\bar{a}} K^{\bar{a}a}|_{\phi_0,\bar{\phi}_0}$), eq. (2.10) implies that $\vec{U}$ is a Killing vector of the Kähler metric $\mathbb{K}$ when we restrict ourself to the curve $\phi(\lambda), \bar{\phi}(\bar{\lambda})$, that is, the Lie derivative vanishes on this curve,

$$\mathcal{L}_{\vec{U}} \mathbb{K}|_{\phi(\lambda),\bar{\phi}(\bar{\lambda})} = 0\ . \tag{2.14}$$

Let us then consider a variation of the fields ϕ^a around the vacuum $\delta\phi^a = \bar{W}_{\bar{a}} K^{\bar{a}a}|_{\phi_0,\bar{\phi}_0}\delta\lambda + \varphi^a \delta\lambda^{n+1}$. The leading term of the variation of V for small $\delta\lambda$ must be positive whatever the choice of the direction φ^a is.

For $1 \leq k \leq n$, the k-th order of variation of V in $\delta\lambda$ reads

$$\delta^k V = \sum_{i=0}^{k} \frac{\delta\lambda^i \delta\bar{\lambda}^{k-i}}{i!(k-i)!} \frac{d^{k-i}}{d\bar{\lambda}^{k-i}} \bar{W}_{\bar{a}} K^{\bar{a}a} \frac{d^i}{d\lambda^i} W_a|_{\phi_0,\bar{\phi}_0} = 0\,, \tag{2.16}$$

by use of condition (2.10) and recurrence relation (2.15). Furthermore, for $0 \leq k \leq n$, the $(n+k+1)$-th order reads

$$\begin{aligned}
\delta^{n+k+1} V \;=\; & \sum_{i=0}^{n+k+1} \frac{\delta\lambda^i \delta\bar{\lambda}^{n+k-i+1}}{i!(n+k-i+1)!} \frac{d^{n+k-i+1}}{d\bar{\lambda}^{n+k-i+1}} \bar{W}_{\bar{a}} K^{\bar{a}a} \frac{d^i}{d\lambda^i} W_a|_{\phi_0,\bar{\phi}_0} \\
& + 2\mathrm{Re}\left\{ \sum_{i=0}^{k} \frac{\delta\lambda^{n+i+1} \delta\bar{\lambda}^{k-i}}{i!(k-i)!} \frac{d^{k-i}}{d\bar{\lambda}^{k-i}} \bar{W}_{\bar{a}} \varphi^b \partial_b (K^{\bar{a}a} \frac{d^i}{d\lambda^i} W_a)|_{\phi_0,\bar{\phi}_0} \right\} \\
= \; & 2\mathrm{Re}\left\{ \delta\lambda^{n+k+1} \left[\frac{1}{(n+k+1)!} \bar{W}_{\bar{a}} K^{\bar{a}a} \frac{d^{n+k+1}}{d\lambda^{n+k+1}} W_a|_{\phi_0,\bar{\phi}_0} \right.\right. \\
& \left.\left. + \frac{1}{k!} \bar{W}_{\bar{a}} K^{\bar{a}a} \varphi^b \partial_b \frac{d^k}{d\lambda^k} W_a|_{\phi_0,\bar{\phi}_0} + \delta^{k0} \bar{W}_{\bar{a}} \varphi^b \partial_b K^{\bar{a}a} W_a|_{\phi_0,\bar{\phi}_0} \right] \right\} = 0\,.
\end{aligned} \tag{2.17}$$

The last term vanishes as a consequence of eqs. (2.7), (2.9). The remaining terms must be all zero since, if one of them were not, the leading order in $\delta\lambda$ would be of the form $\mathrm{Re}(\delta\lambda^{n+k+1})$, which takes negative values for some $\delta\lambda$. Hence,

$$\frac{1}{(n+k+1)!} \bar{W}_{\bar{a}} K^{\bar{a}a} \frac{d^{n+k+1}}{d\lambda^{n+k+1}} W_a|_{\phi_0,\bar{\phi}_0} + \frac{1}{k!} \bar{W}_{\bar{a}} K^{\bar{a}a} \varphi^b \partial_b \frac{d^k}{d\lambda^k} W_a|_{\phi_0,\bar{\phi}_0} = 0\,, \qquad \forall\, 0 \leq k \leq n\,. \tag{2.18}$$

Therefore, since φ^a is an arbitrary vector, $\bar{W}_{\bar{a}} K^{\bar{a}a} \partial_b \frac{d^k}{d\lambda^k} W_a|_{\phi_0,\bar{\phi}_0}$ must be itself equal to zero. Then, using that $\partial_b \frac{d^k}{d\lambda^k} W_a = \partial_a \frac{d^k}{d\lambda^k} W_b$ and taking $k = n$ gives the result

$$\frac{d^{n+1}}{d\lambda^{n+1}} W_a|_{\phi_0} = 0\,, \tag{2.19}$$

so the recurrence condition is verified one step further. This, together with the fact that the recurrence condition (2.15) is true for $n = 1$, implies that

$$\frac{d^n}{d\lambda^n} W_a|_{\phi_0} = 0\,, \qquad \forall\, n > 0\,, \tag{2.20}$$

and then W_a is constant along the curve eq. (2.12),

$$W_a(\phi(\lambda), \bar{\phi}(\bar{\lambda})) = W_a(\phi_0, \bar{\phi}_0)\,, \tag{2.21}$$

with λ any complex number. Since $K^{\bar{a}a}$ is also constant along this curve (see eq. (2.13)), it is trivial to check that the same happens with the potential $V = \bar{W}_{\bar{a}} K^{\bar{a}a} W_a$.

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In summary, we have shown that when the conditions (2.6)–(2.10) are satisfied, the potential V is degenerate along a one-dimensional sub-manifold,

$$V(\phi(\lambda), \bar{\phi}(\bar{\lambda})) = V(\phi_0, \bar{\phi}_0) \ , \tag{2.22}$$

with the curve $\phi(\lambda), \bar{\phi}(\bar{\lambda})$ given by

$$\phi^a(\lambda) = \bar{W}_{\bar{a}} K^{\bar{a}a}|_{\phi_0,\bar{\phi}_0}(\lambda - \lambda_0) + \phi_0^a \,, \qquad \bar{\phi}^{\bar{a}}(\bar{\lambda}) = K^{\bar{a}a} W_a|_{\phi_0,\bar{\phi}_0}(\bar{\lambda} - \bar{\lambda}_0) + \bar{\phi}_0^{\bar{a}} \ . \tag{2.23}$$

2.2 Models with one pseudomoduli

In this section we will use the results obtained in section 2.1 to find the most general non-canonical Kähler consistent with a particular type of superpotentials recently analized by Shih [12] in the case of canonical Kähler, which are a generalization of the original O'Raifeartaigh models. These R-symmetric superpotentials can be written as

$$W = fX + \frac{1}{2}(B_{ij} + XA_{ij})\phi^i\phi^j, \tag{2.24}$$

where f is a complex constant, and A and B are symmetric complex matrices satisfying $det(B) \neq 0$ (see below). In order for the superpotential to be R-symmetric, we require

$$A_{ij} \neq 0 \Rightarrow R(\phi^i) + R(\phi^j) = 0 \,, \qquad B_{ij} \neq 0 \Rightarrow R(\phi^i) + R(\phi^j) = 2 \ . \tag{2.25}$$

In the case of canonical Kähler [12], supersymmetry is broken in this model and a non-supersymmetric minimum $V_0 = |f|^2$ is given by

$$\phi^i = 0 \,, \qquad X \text{ arbitrary} \ . \tag{2.26}$$

Therefore, the field X become a modulus parameterizing the vacua manifold. This planar direction is lifted by quantum corrections so X is called a pseudomodulus.

Let us see now how the Kähler potential should be in order to maintain the degeneracy at tree level along the curve (2.26). From condition (2.10) (or, equivalently, condition (2.13)), the components $K^{\bar{a}a}$ must be constant along the curve, and then

$$\partial_X K_{a\bar{a}}(X, \phi^i = 0) = 0 \,, \qquad \partial_a K_{X\bar{a}}(X, \phi^i = 0) = 0 \ . \tag{2.27}$$

Concerning condition (2.9), this can be written as

$$K^{\bar{X}i}(B_{ij} + XA_{ij}) = 0 \ . \tag{2.28}$$

As shown by Shih [12], the constraints due to R-symmetry (2.25) imply that $det(B+XA) = det(B) \neq 0$. Therefore, $K^{\bar{X}i}(X, \phi^i = 0)$ must vanish, leading to

$$K_{X\bar{i}}(X, \phi^i = 0) = 0 \,, \qquad K_{i\bar{X}}(X, \phi^i = 0) = 0 \ . \tag{2.29}$$

The last condition to impose, coming from eq. (2.7), is $W_{\bar{a}}\partial_b K^{\bar{a}a} W_a(X, \phi^i = 0) = |f|^2 \partial_b K^{\bar{X}X}(X, \phi^i = 0) = 0$. However, it is easy to show that in our case this condition is already implied by (2.27) and (2.29). From $K^{\bar{X}a}K_{a\bar{X}} = 1$ we obtain

$$\partial_b K^{\bar{X}X} K_{X\bar{X}} + \partial_b K^{\bar{X}i} K_{i\bar{X}} + K^{\bar{X}X}\partial_b K_{X\bar{X}} + K^{\bar{X}i}\partial_b K_{i\bar{X}} = 0 \ . \tag{2.30}$$

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When we evaluate this equation in $\phi^i = 0$, the second and fourth terms vanish due to (2.29), while the third one due to (2.27), this leaving us with the desired result.

In summary, the conditions we have to impose on the Kähler potential to have degeneracy in $\phi^i = 0$ for arbitrary X are

$$\partial_{\bar{a}}\partial_{aX}K(X,\phi^i = 0) = 0\,, \qquad \partial_{\bar{i}}\partial_X K(X,\phi^i = 0) = 0\,. \tag{2.31}$$

If we consider an expansion of K in powers of X and $\bar{X}$

$$K(X,\phi^i,\bar{X},\bar{\phi}^{\bar{j}}) = \sum_{m,n=0}^{\infty} f_{mn}(\phi^i,\bar{\phi}^{\bar{j}})X^m\bar{X}^n, \qquad f_{nm} = (f_{mn})^*\,, \tag{2.32}$$

then, conditions (2.31) can be expressed as

$$\begin{aligned}
\partial_{\bar{j}}f_{m0}(0) &= \partial_i\partial_{\bar{j}}f_{m0}(0) = 0 && m > 0\\
f_{11}(0) &= \text{const.}\,, \qquad \partial_i f_{11}(0) = \partial_{\bar{j}}f_{11}(0) = \partial_i\partial_{\bar{j}}f_{11}(0) = 0 &&\\
f_{mn}(0) &= \partial_i f_{mn}(0) = \partial_{\bar{j}}f_{mn}(0) = \partial_i\partial_{\bar{j}}f_{mn}(0) = 0 && m,n \geq 1, m+n > 2.
\end{aligned} \tag{2.33}$$

The simplest example of Kähler potential leading to the required degeneracy consists in imposing that equation (2.33) be valid not only in $\phi^i = 0$ but for any ϕ^i. In this case, K can be written as

$$K = cX\bar{X} + C(\phi^i,\bar{\phi}^{\bar{j}})\,, \tag{2.34}$$

with c any real constant (that can trivially be taken to 1 by a rescaling of X).

2.3 Models with more pseudo-moduli

Based on the result eq. (2.34), it is easy to propose a model possessing several pseudomoduli. An obvious generalization of the non-canonical Kähler potential (2.34) is given by

$$K = X^\alpha \delta_{\alpha\bar{\alpha}}\bar{X}^{\bar{\alpha}} + C(\phi^i,\bar{\phi}^{\bar{j}})\,, \tag{2.35}$$

where $\alpha,\beta,\ldots = 1,\ldots,N_X$ label the N_X fields X^α that appear in K in a canonical form, while $i,j,\ldots = 1,\ldots,N_\phi$ label the N_ϕ fields ϕ^i with non-canonical structure. Thus, the Kähler potential (2.35) defines a (regular and positive defined) metric in field space with matricial form

$$K_{a\bar{a}} = \partial_a\partial_{\bar{a}}K = \begin{pmatrix} \delta_{\alpha\bar{\alpha}} & 0 \\ 0 & C_{i\bar{i}} \end{pmatrix}\,, \tag{2.36}$$

where we have defined the $N_\phi \times N_\phi$ matrix

$$C_{i\bar{j}} = \partial_i\partial_{\bar{j}}C\,. \tag{2.37}$$

Concerning the superpotential, inspired in the generalized O'Raifeartaigh models, we consider superpotentials with the form [7]

$$W = f^\alpha X^\alpha + \frac{1}{2}(B_{ij} + X^\alpha A^\alpha_{ij})\phi^i\phi^j\,, \tag{2.38}$$

for which the scalar potential $V = \bar{W}_{\bar{a}} K^{\bar{a}a} W_a$, reads

$$V = \ |f^\alpha + \frac{1}{2} A^\alpha_{ij} \phi^i \phi^j|^2 + \bar{\phi}^{\bar{i}} (\bar{B} + \bar{X}^{\bar{\alpha}} \bar{A}^{\bar{\alpha}})_{\bar{i}\bar{j}} C^{\bar{j}j} (B + X^\alpha A^\alpha)_{ji} \phi^i \ . \tag{2.39}$$

In order for these models to be R-symmetric, K must be R-symmetric and the R-charges of the fields should be such that

$$R(X^\alpha) = 2\,, \qquad A^\alpha_{ij} \neq 0 \Rightarrow R(\phi^i) + R(\phi^j) = 0\,, \qquad B_{ij} \neq 0 \Rightarrow R(\phi^i) + R(\phi^j) = 2\,. \tag{2.40}$$

Analogously to the case of the superpotential (3.1), these models break supersymmetry. In fact, the equations for a supersymmetry vacuum are

$$\begin{aligned} f^\alpha + \frac{1}{2} A^\alpha_{ij} \phi^i \phi^j &= 0 \\ (B_{ij} + X^\alpha A^\alpha_{ij}) \phi^j &= 0\,. \end{aligned} \tag{2.41}$$

Similarly to the case of one pseudomoduli, conditions (2.40) imply that $det(B + X^\alpha A^\alpha) = det(B) \neq 0$. Therefore, equations (2.41) are not compatible. Besides, it is clear that a non supersymmetric minimum $V = |f_\alpha|^2$ appears at

$$\phi^i = 0\,, \qquad X^\alpha \text{ arbitrary}\,, \tag{2.42}$$

implying that the fields X^α are pseudomoduli parameterizing the vacua manifold.

In this general case, we have a N_X-dimensional manifold of non-supersymmetric vacua, parameterized by X^α. Then, in order to generalize this Kähler potential, we can think of a more general dependence on the X^α fields

$$K = k(X^\alpha, \bar{X}^{\bar{\alpha}}) + C(\phi^i, \bar{\phi}^{\bar{j}})\,, \tag{2.43}$$

with the corresponding tree-level lifting

$$V_{\phi=0}(X^\alpha, \bar{X}^{\bar{\alpha}}) = f_\alpha k^{\alpha\bar{\alpha}} f_{\bar{\alpha}}\,, \tag{2.44}$$

when $k^{\alpha\bar{\alpha}} \neq \delta^{\alpha\bar{\alpha}}$. This is nothing but the generalization of (2.4), which is the 1-dimensional case. This "tree-level moduli lifting" case might be studied on general grounds by a classification of the different $k^{\alpha\bar{\alpha}}$ metrics (some examples in the 1-dimensional case were computed in [14]). Moreover, this would be a starting point to study new terms mixing the X and ϕ fields. In this paper we do not consider these possibilities.

3. O'Raifeartaigh models

In this section we consider quiral models recently introduced in [12], which are a generalization of the original O'Raifeartaigh model [4]. We have already considered them in section 2.2, but we define them again in order for this section to be self-contained. The superpotential for this theory is

$$W = fX + \frac{1}{2} B_{ij} \phi^i \phi^j + \frac{1}{2} X A_{ij} \phi^i \phi^j\,, \tag{3.1}$$

where f is a complex constant and A and B are symmetric complex matrices. The matrix B satisfies $det(B) \neq 0$ and A and B have non-zero entries only when

$$A_{ij} \neq 0 \Rightarrow R(\phi^i) + R(\phi^j) = 0\,, \qquad B_{ij} \neq 0 \Rightarrow R(\phi^i) + R(\phi^j) = 2\,, \tag{3.2}$$

so W has a definite R-charge $R(W) = 2$. The susy vacua conditions for this theory read

$$f + \frac{1}{2} A_{ij}\phi^i\phi^j = 0 \tag{3.3}$$

$$(XA + B)_{ij}\phi^j = 0\,. \tag{3.4}$$

Because of R-symmetry, A and B adopt the following matricial form

$$A = \begin{pmatrix} 0 & & A_1 & 0 \\ & \cdots & & \\ A_1^T & & & \\ & & & \\ 0 & & & 0 \end{pmatrix}, \qquad B = \begin{pmatrix} 0 & & & B_1 \\ & & B_2 & \\ & \cdots & & \\ & B_2^T & & \\ B_1^T & & & 0 \end{pmatrix} \tag{3.5}$$

in some field basis. As shown by Shih [12], this particular structure for the matrices implies $det(XA + B) = det(B)$. Then, as we have taken this to be non-zero, the only solution for (3.4) is $\phi_i = 0\ \forall i$, so (3.3) can never be satisfied. Susy is therefore broken in this model, and a non-supersymmetric minimum is given by

$$\phi^i = 0\,, \qquad X \text{ arbitrary}\,, \qquad V_0 = |f|^2\,. \tag{3.6}$$

The above considerations are independent of the Kähler potential. For K we consider that obtained in (2.34), which can be written without loss of generality as

$$K = \bar{X}X + C(\phi,\bar{\phi})$$

$$C(\phi,\bar{\phi}) = \phi^i C_{i\bar{j}}\bar{\phi}^{\bar{j}} + \ldots\,. \tag{3.7}$$

Here we have taken the c parameter in (2.34) to be $c = 1$ by rescaling the field X. $C^{i\bar{j}}$ is an hermitic matrix that satisfies $C^{i\bar{j}} \neq 0 \Rightarrow R(\phi^i) + R(\bar{\phi}^{\bar{j}}) = 0$, and "$\ldots$" are cubic or higher terms. In the basis in which A and B take the form (3.5), C has a diagonal-block form with blocks of fields having the same R-charge.

It is easy to see that performing a change of the field basis (not necessarily a unitary transformation), the quadratic part of the Kähler can be taken to have a canonical form, leaving the superpotential with the same structure as in (3.1)

$$K = \bar{X}X + D(\phi,\bar{\phi})\,, \tag{3.8}$$

$$D(\phi,\bar{\phi}) = \phi^i \delta_{i\bar{j}}\bar{\phi}^{\bar{j}} + \ldots\,, \tag{3.9}$$

$$W = fX + \frac{1}{2}M_{ij}\phi^i\phi^j + \frac{1}{2}XN_{ij}\phi^i\phi^j\,. \tag{3.10}$$

Here we have written the transformed fields and introduced new matrices N and M, which are generic symmetric matrices with the same form of A and B in (3.5) respectively. Then, after this change of basis, we are left (in a neighborhood of the $\phi^i = 0$ vacua) with a theory with superpotential (3.10) and canonical Kähler potential.

Although we have diagonalized the quadratic dependence of the Kähler potential, one can not get rid of its curvature and then, those properties depending on cubic and higher order terms will change. Interestingly, the stability of the $\phi^i = 0$ pseudo-moduli space in the ϕ^i direction is not affected by cubic or higher order terms. The reason for this is that the mass squared matrix for bosons (see eq. (A.9) in appendix) in this vacua only depends on $K^{a\bar{a}}$ and not on its derivatives.

One important feature that arises when the Kähler is non-minimal is that the mass squared matrices (see eqs. (A.9), (A.10) in the appendix) get modified in such a way that their eigenvalues split, even at tree level. The so-called supertrace theorem [15] generically implies the existence of a supersymmetric particle lighter than its ordinary partner, and then the paradigm for constructing realistic SUSY theories is to assume that the SUSY-breaking sector has no renormalizable tree level couplings with the observable sector. The latter theorem follows from the properties of renormalizability that force the kinetic terms to have a canonical form. This is not our case, as we are considering effective low-energy theories which not necessarily have a canonical Kähler potential, this leading to the important phenomenological consequence mentioned above of mass splitting at tree-level. The mass-squared matrices of this model (3.1), (3.7) in the vacuum $\phi^i = 0$ are

$$\begin{aligned} \mathcal{M}_F^2 &= (\hat{B} + X\hat{A})^2 \\ \mathcal{M}_B^2 &= (\hat{B} + X\hat{A})\ \hat{C}\ (\hat{B} + X\hat{A}) + f\hat{A}\,, \end{aligned} \tag{3.11}$$

where we have defined

$$\hat{A} = \begin{pmatrix} 0 & A \\ A^\dagger & 0 \end{pmatrix}, \qquad \hat{B} = \begin{pmatrix} 0 & B \\ B^\dagger & 0 \end{pmatrix}, \qquad \hat{C} = \begin{pmatrix} C^{-1} & 0 \\ 0 & C^{-1} \end{pmatrix}, \tag{3.12}$$

being A and B the matrices (3.5), and C the matrix defined in (2.37) which has a diagonal block form with blocks of fields having the same R-charge.

Following Ferretti's approach [7], we now look for possible restrictions to the Kähler potential by demanding this theory to posses runaway directions. As (3.3) and (3.4) are incompatible, we look for a compatible subset of equations. In fact, classifying (3.4) acording to their R-charge we have

$$(XA + B)_{ij}\phi^j = 0\,, \qquad R(\phi^i) < 2 \tag{3.13}$$

$$(XA + B)_{kj}\phi^j = 0\,, \qquad R(\phi^k) = 2 \tag{3.14}$$

$$(XA + B)_{mj}\phi^j = 0\,, \qquad R(\phi^m) > 2\,, \tag{3.15}$$

and it was shown in [7] that for generic R-charge assignments it is always possible to solve (3.3)–(3.13)–(3.14). We call the solutions to these equations X', ϕ'^i. The potential for this particular configuration of fields reads

$$V_0' = \sum_{R(\phi^m)>2\ \&\ R(\phi^n)>2} C'^{\bar{m}n}\,(X'A + B)_{ni}\big(\bar{X}'\bar{A} + \bar{B}\big)_{\bar{j}\bar{m}}\phi'^i\bar{\phi}'^{\bar{j}}\ . \tag{3.16}$$

Interestingly, by looking at (3.3)–(3.13)–(3.14), we see that a continuously connected range of solutions parameterized by a parameter δ is obtained from every solution X', ϕ'^i

$$\phi^i(\delta) = \delta^{-R(\phi^i)}\phi'^i\,, \qquad X(\delta) = \delta^{-2}X'\ . \tag{3.17}$$

For these fields, the potential now reads

$$\begin{aligned} V_0(\delta) &= \sum_{R(\phi^m)>2\ \&\ R(\phi^n)>2} (C(\delta))^{\bar{m}n}(X(\delta)A+B)_{ni}(\bar{X}(\delta)\bar{A}+\bar{B})_{\bar{j}\bar{m}}\phi^i(\delta)\bar{\phi}^{\bar{j}}(\delta) \qquad (3.18)\\ &= \sum_{R(\phi^m)>2\ \&\ R(\phi^n)>2} (C(\delta))^{\bar{m}n}(X'A+B)_{ni}(\bar{X}'\bar{A}+\bar{B})_{\bar{j}\bar{m}}\phi'^i\bar{\phi}'^{\bar{j}}\delta^{-4+R(\phi^m)+R(\phi^n)}\ . \end{aligned}$$

This potential slopes to zero when

$$\lim_{\delta\to 0}\delta^{R(\phi^m)+R(\phi^n)-4}(C(\delta))^{\bar{m}n} = 0\,, \qquad \forall n,m/R(\phi^m)>2\ \&\ R(\phi^n)>2\ . \tag{3.19}$$

This doesn't seem a hard restriction provided that already $\delta^{R(\phi^m)+R(\phi^n)-4} \to 0$ when $\delta \to 0$. We have found a sufficient condition on the Kähler potential in order for the theory to have runaway behavior. Moreover, the Kähler can induce runaway behavior, even if the canonical theory has no runaway directions.

Recently, strongly convincing arguments have been given that we happen to live in a metastable vaccum [3]. Thus, in order to construct viable phenomenological models, besides the SUSY-breaking minimum, these models must have runaway directions and/or supersymmetric vacua. Moreover, the notion of meta-stable states is meaningful only when they are parametrically long lived since, phenomenologically, we would like the lifetime of our meta-stable state to be longer than the age of the Universe. It is therefore important for us to have the possibility of modifying the landscape of vacua by adjusting the parameters of the Kähler potential, since in this way one can guarantee the longevity of the meta-stable state.

4. R-symmetry breaking

For the O'Raifeartaigh models of the previous sections (3.1), (3.7)

$$W = fX + \frac{1}{2}B_{ij}\phi^i\phi^j + \frac{1}{2}XA_{ij}\phi^i\phi^j\,, \qquad K = \bar{X}X + C(\phi,\bar{\phi})\,, \tag{4.1}$$

where X is the coordinate parameterizing the pseudomoduli space. The fact that X is a coordinate of the one-dimensional vacua manifold requires analysis beyond tree-level. Thus, we expect that radiative corrections to the scalar potential will determine the vacuum properties dynamically. Moreover, these corrections must respect the symmetries of the original theory, so we can already anticipate their form

$$V_{\text{eff}}(|X|^2) = V_0 + m_X^2|X|^2 + \mathcal{O}(|X|^4)\ . \tag{4.2}$$

It has an extremum at $X = 0$ so it lifts the classical vacuum degeneracy. Moreover, as it is shown below, m_X^2 can take values which are not necessarily positive.

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The first order in the loop expansion of $V_{\rm eff}$ is given by the formula (see eq. (A.21) in appendix)

$$V_{\rm eff}^{(1)} = \frac{1}{64\pi^2}{\rm Tr}\left(\tilde{\mathcal{M}}_B^4\left[\log\left(\frac{\tilde{\mathcal{M}}_B^2}{\Lambda^2}\right)-\frac{1}{2}\right]-\tilde{\mathcal{M}}_F^4\left[\log\left(\frac{\tilde{\mathcal{M}}_F^2}{\Lambda^2}\right)-\frac{1}{2}\right]+\frac{\Lambda^2}{2}\left(\tilde{\mathcal{M}}_B^2-\tilde{\mathcal{M}}_F^2\right)\right), \tag{4.3}$$

where

$$\tilde{\mathcal{M}}_F^2 = K^{-1/2}\mathcal{M}_F K^{-1}\mathcal{M}_F K^{-1/2}, \qquad \tilde{\mathcal{M}}_B^2 = K^{-1/2}\mathcal{M}_B^2 K^{-1/2}. \tag{4.4}$$

There is also an additional term proportional to Λ^4, which we omit here because it is constant. Our aim is to derive a general formula for m_X^2 in the one-loop approximation as was done in [12] but in the non-canonical model proposed in the previous sections. This will tell us wether the X filed acquires a VEV or not. If it does ($m_X^2 < 0$) then, as $R(X) = 2$, R-symmetry is broken. Otherwise, R-symmetry remains unbroken in this vacuum.

For the O'Raifeartaigh models the tilde - mass matrices read

$$\begin{aligned}\tilde{\mathcal{M}}_F^2 &= \hat{C}^{1/2}\,(\hat{B}+X\hat{A})\,\hat{C}\,(\hat{B}+X\hat{A})\,\hat{C}^{1/2}\\ \tilde{\mathcal{M}}_B^2 &= \hat{C}^{1/2}\,(\hat{B}+X\hat{A})\,\hat{C}\,(\hat{B}+X\hat{A})\,\hat{C}^{1/2}+f\hat{A},\end{aligned} \tag{4.5}$$

where $\hat{A}$, $\hat{B}$ and $\hat{C}$ have been defined in (3.12), and the following identities hold

$${\rm Tr}(\tilde{\mathcal{M}}_B^2-\tilde{\mathcal{M}}_F^2)=0, \qquad {\rm Tr}\frac{\partial^2}{\partial X^2}(\tilde{\mathcal{M}}_B^4-\tilde{\mathcal{M}}_F^4)\,|_{X=0}=0. \tag{4.6}$$

This is a very interesting result because it implies that one-loop corrections to the scalar potential are not quadratic, but logarithmic in Λ. And also, m_X^2 is independent of Λ. These two features are always true (independently of the superpotential) in canonical Kähler models. Here, it is true due to the form these matrices adopt because of the R-symmetry.

Then, in this case, the effective potential can be written as

$$V_{\rm eff}^{(1)} = -\frac{1}{32\pi^2}{\rm Tr}\int_0^\Lambda dv\; v^5\;\left(\frac{1}{v^2+\tilde{\mathcal{M}}_B^2}-\frac{1}{v^2+\tilde{\mathcal{M}}_F^2}\right), \tag{4.7}$$

and we can substitute (4.5) in (4.7) to obtain an expression for $m_X^2 = \frac{1}{2}\frac{\partial^2 V_{\rm eff}^{(1)}}{\partial X^2}\,|_{X=0}$, which is Λ-independent

$$\begin{aligned}m_X^2 = \frac{1}{16\pi^2}{\rm Tr}\int_0^\infty dv\,\Bigg[v^3\frac{1}{v^2+\mathcal{B}^2+f\hat{A}}\left(\mathcal{A}^2-\frac{1}{2}\{\mathcal{A},\mathcal{B}\}\frac{1}{v^2+\mathcal{B}^2+f\hat{A}}\{\mathcal{A},\mathcal{B}\}\right)\\ -\frac{1}{v^2+\mathcal{B}^2}\left(\mathcal{A}^2-\frac{1}{2}\{\mathcal{A},\mathcal{B}\}\frac{1}{v^2+\mathcal{B}^2}\{\mathcal{A},\mathcal{B}\}\right)\Bigg].\end{aligned} \tag{4.8}$$

Here we have integrated by parts and defined

$$\mathcal{A} = \hat{C}^{1/2}\hat{A}\hat{C}^{1/2}, \qquad \mathcal{B} = \hat{C}^{1/2}\hat{B}\hat{C}^{1/2}. \tag{4.9}$$

Now, defining

$$\mathcal{F}(v) = (v^2+\mathcal{B}^2)^{-1}f\mathcal{A}, \qquad \mathcal{G}(v) = (v^2+\mathcal{B}^2)^{-1}f\hat{A}, \tag{4.10}$$

we can write

$$m_X^2 = M_1^2 - M_2^2 \,, \tag{4.11}$$

where

$$M_1^2 = \frac{1}{16\pi^2 f^2}\mathrm{Tr}\int_0^\infty dv\ v^5 \mathcal{F}^2 \frac{\mathcal{G}^2}{1-\mathcal{G}^2} \tag{4.12}$$

$$M_2^2 = \frac{1}{2}\frac{1}{16\pi^2 f^2}\mathrm{Tr}\int_0^\infty dv\ v^3 \left(\frac{\mathcal{G}}{1-\mathcal{G}^2}\{\mathcal{F},\mathcal{B}\}\right)^2 \,. \tag{4.13}$$

It is easy to see that if only fields with R-charge $R = 0, 2$ assignments are present, then $M_2^2 = 0$ and $M_1^2 > 0$, so R-symmetry is not broken as it happens in [12]. This suggests that generic R-charge assignments should be made to quiral models in order for the R-symmetry to be broken. In the next section we consider a model of this kind.

Let us end this section with a brief disclaimer about explicit R-symmetry breaking and some comments on the phenomenological consequence of considering non-canonical Kähler in relation to R-symmetry. Explicit R-symmetry in these models have been studied in [8, 9]. In these works the Kähler is canonical and R-symmetry breaking terms are added to the superpotential, leading to the appearance of supersymmetric vacua, in agreement with the Nelson-Seiberg argument [6]. As expected, in the limit of small R-symmetry breaking the susy vacua can be pushed sufficiently far from the origin of field space, thus making the metastable vacua parametrically long-lived. Trying to repeat this procedure by breaking R-symmetry from the Kähler potential fails. The reason for this is that the conditions for supersymmetry breaking depend only on the superpotential W and not on the form of the Kähler potential.

As stated by Nelson and Seiberg [6], it is a necessary condition for SUSY-breaking in generic models to have an R-symmetry, and a sufficient condition that R-symmetry is spontaneously broken. From a phenomenological point of view this fact is problematic because an unbroken R-symmetry forbids Majorana gaugino masses, and having an exact but spontaneously broken R-symmetry leads to a light R-axion. Let us mention how we get rid of this apparent problem. First of all lets us comment that our model is not generic, so the Nelson-Seiberg argument is not applicable. Therefore, we could in principle be able to break R-symmetry explicitly from the superpotential without restoring SUSY (see an example in [9]). We have not explored this possibility, instead we have considered two other cases. One in which (following [12]) we have an R-symmetry which can be spontaneously broken. In this case, we can expect that including gravity will make the R-axion sufficiently massive [16]. Another case, commented in the previous paragraph, in which R-symmetry is explicitly broken in the Kähler potential. This possibility is free from the above problems, since R-symmetry breaking does not induce SUSY vacua, and we have no goldstone boson because the symmetry is explicitly broken. Another possibility for spontaneous R-symmetry breaking could be choosing an R-symmetric Kähler potential generating a tree-level lifting of the moduli space giving rise to a non-R-symmetric minimum.

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5. Shih model with non-canonical Kähler

We have shown in the previous section that in order for the O'Raifeartaigh models (3.8)–(3.10) to have R-symmetry broken, there must be in the theory at least one field with R-charge different from 0 or 2. A model of this kind was proposed in [12]. The model has superpotential

$$W = \lambda X \phi_1 \phi_2 + m_1 \phi_1 \phi_3 + \frac{1}{2} m_2 \phi_2^2 + f X \,, \tag{5.1}$$

and in our case the Kähler potential reads

$$K = \bar{X} X + C(\bar{\phi}^{\bar{j}}, \phi_j) \ . \tag{5.2}$$

By rotating the phases of all the fields, the couplings can be taken to be real and positive, without loss of generality. In order for the theory to be R-symmetric, the R-charge assignments must be $R(X) = 2$, $R(\phi_1) = -1$, $R(\phi_2) = 1$ and $R(\phi_3) = 3$. Notice that because the R-charge assignments are all different, the Kähler potential depends on the fields in the form $\phi_i \bar{\phi}^i$. Then, the transformation that takes the quadratic part of the Kähler to its canonical form consists only of a rescaling of the fields. This rescaling, together with a redefinition of the constants, leaves the superpotential invariant. In other words, this model is (near $\phi = 0$) nothing but the Shih model with redefined constants. We review some properties of this model to see what can be changed by considering non-canonical Kähler.

The extrema of the potential consists of the pseudo-moduli space

$$\phi_i = 0 \,, \qquad \forall X \qquad \longrightarrow \qquad V_0 = f^2 \ . \tag{5.3}$$

This is the only extrema if the Kähler is canonical. Depending on the explicit form of $C(\bar{\phi}^j \phi_j)$ other extrema can appear. The pseudo-moduli space is a minimum of the potential when

$$|X| < \frac{c_1}{2} \left(1 - \frac{f\lambda}{c_2 c_3 m_1 m_2}\right) / \left(\frac{f\lambda^2}{m_1^2 m_2}\right) \ , \tag{5.4}$$

where

$$c_i \equiv \left(\frac{\partial C}{\partial(\phi_i \bar{\phi}^i)}\right)^{-1}_{\phi=0} \ , \tag{5.5}$$

otherwise some eigenvalues of the mass squared matrix become tachyonic. This pseudo-moduli space is only a local minima of the potential provided there is a runaway direction

$$X = \left(\frac{m_1^2 m_2 \phi_3^2}{f\lambda^2}\right)^{1/3} \ , \quad \phi_1 = \left(\frac{m_2 f^2}{\lambda^2 m_1 \phi_3}\right)^{1/3} \ , \quad \phi_2 = -\left(\frac{m_1 f \phi_3}{\lambda m_2}\right)^{1/3} \ , \tag{5.6}$$

as long as (3.19) is satisfied

$$\lim_{\phi_3 \to \infty} \phi_3^{-2/3} \left(\frac{\partial^2 C}{\partial \phi_3 \partial \bar{\phi}^3}\right)^{-1} = 0 \ . \tag{5.7}$$

Notice that the direction of the runaway in field space is the same as its canonical counterpart, but the value of the scalar potential evaluated on this direction is modified. This runaway direction can be parameterized by X, and along this direction the potential takes the values

$$V_{RA}(|X|) = \left(\frac{\partial^2 C}{\partial\phi_3 \partial\bar{\phi}^3}\right)^{-1}(|X|)\ \frac{m_1^2 m_2 f}{\lambda^2 |X|}\ . \tag{5.8}$$

The value of $|X|$ for which $V_{RA} = V_0$ gives an estimate of the vacuums life-time. It can be taken parametrically long-lived, and moreover, the Kähler potential can change the lifetime.

The A, B and C^{-1} matrices in (2.36), (3.5) are

$$A = \begin{pmatrix} 0 & \lambda & 0 \\ \lambda & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\ , \quad B = \begin{pmatrix} 0 & 0 & m_1 \\ 0 & m_2 & 0 \\ m_1 & 0 & 0 \end{pmatrix}\ , \quad C^{-1} = \begin{pmatrix} c_1 & 0 & 0 \\ 0 & c_2 & 0 \\ 0 & 0 & c_3 \end{pmatrix}\ . \tag{5.9}$$

As a test of the expressions (4.12)–(4.13) we have explicitly evaluated them for this non-canonical model and further compared the results with the rescaled results of [12]. The calculation with the formulas we derived is

$$M_1^2 = \frac{c_1^2 c_2^2}{4\pi^2 f^2}\int_0^\infty dv\ v^5\ \frac{f^4\lambda^4}{(v^2+\tilde{m}_1^2)(v^2+\tilde{m}_2^2)\left((v^2+\tilde{m}_1^2)(v^2+\tilde{m}_2^2) - f^2\lambda^2\right)} \tag{5.10}$$

$$M_2^2 = \frac{c_1^2 c_2^2}{2\pi^2 f^2}\int_0^\infty dv\ v^3\ \frac{f^4\lambda^4\tilde{m}_2^2}{\left((v^2+\tilde{m}_1^2)(v^2+\tilde{m}_2^2) - f^2\lambda^2\right)^2} \tag{5.11}$$

where we defined $\tilde{m}_1 = c_1 c_3 m_1$ and $\tilde{m}_2 = c_2 m_2$, and this can be rewritten as

$$M_1^2 = \frac{c_1^2 c_2^2 \lambda^2 \tilde{m}_1^2 r^2 y^2}{4\pi^2}\int_0^\infty dv\ v^5\ \frac{1}{(v^2+1)(v^2+r^2)\left((v^2+1)(v^2+r^2) - y^2 r^2\right)} \tag{5.12}$$

$$M_2^2 = \frac{c_1^2 c_2^2 \lambda^2 \tilde{m}_1^2 r^4 y^2}{2\pi^2}\int_0^\infty dv\ v^3\ \frac{1}{\left((v^2+1)(v^2+r^2) - y^2 r^2\right)^2} \tag{5.13}$$

where we have defined $y = \frac{\lambda f}{\tilde{m}_1 \tilde{m}_2}$ and $r = \tilde{m}_2/\tilde{m}_1$. Integrating this expression to order $\mathcal{O}(y^2)$ we obtain

$$M_1^2 = \frac{c_1^2 c_2^2 \lambda^2 \tilde{m}_1^2 r^2 y^2}{8\pi^2}\ \frac{r^4 - 4r\log(r) - 1}{(r^2-1)^3} + \mathcal{O}(y^4) \tag{5.14}$$

$$M_2^2 = \frac{c_1^2 c_2^2 \lambda^2 \tilde{m}_1^2 r^4 y^2}{2\pi^2}\ \frac{(r^2+1)\log(r) + 1 - r^2}{(r^2-1)^3} + \mathcal{O}(y^4)\ . \tag{5.15}$$

These expressions are identical to those obtained in [12], although in this case the definition of the parameters y and r depend on the Kähler potential. It was shown in [12] that some r^* exists such that for $r > r^*$ the $m_X^2 = M_1^2 - M_2^2 < 0$, so R-symmetry is broken. A non-canonical Kähler potential cannot change this behavior, although it can change the value of r^*.

6. Summary and discussion

Several aspects of R-symmetry and supersymmetry breaking have been studied in generalized O'Raifeartaigh models with non-canonical Kähler potential. We derived some

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conditions on the Kähler potential in order for the non-supersymmetric vacua to be degenerate at tree-level. This is a common feature of renormalizable models and we show that it is also shared by many non-renormalizable theories.

Once degeneracy is guaranteed for the vacuum at the classical level, the information about the lifting of the flat directions is given by the CW effective potential. We calculated the CW potential for arbitrary quiral non-linear sigma-models, and this allowed us to study the 1-loop quantum corrections to the pseudo-moduli space. This potential has a quadratic and a quartic dependence on the cutoff scale Λ which vanish identically in supersymmetric models with canonical Kähler. In our case the quadratic dependence also vanishes, which can be seen as a consequence of R-symmetry in our model, and the quartic dependence becomes constant in the considered vacuum. Concerning the logarithmic divergent term $\log(\Lambda)$ STr $\mathcal{M}^4$, it can usually be absorbed into the renormalization of the coupling constants appearing in the tree-level vacuum energy in theories with canonical Kähler. It would also be interesting to study if this is the case in our non-renormalizable R-symmetric models. Another interesting fact is that the mass of the flat mode is independent of Λ also due to R-symmetry, this happens in renormalizable models as well. These similarities between R-symmetric models with canonical Kähler potential, and R-symmetric models with non-canonical Kähler potentials require further research. One may wonder if these similarities between models with canonical and non-canonical Kähler are extensive for any R-symmetric superpotential, or if they are only valid in this generalized O'Raifeartaigh model.

The conditions for R-symmetry breaking remain unchanged with respect to those of canonical models. R-symmetry can be broken when generic R-charge assignments to the fields are made, while R-symmetry remains unbroken when only fields with R-charge 0 and 2 are present. In [17], based on the number of fields with 0 and 2 R-charge, more information is obtained about the properties of the model regarding symmetry breaking. It would be interesting to see if a similar analysis can be done in the case of non-canonical models. Another issue to be more thoroughly analyzed concerns the question whether two Kähler potentials exist, such that for a fixed superpotential, R-symmetry is broken in one case and unbroken in the other.

The models we presented can keep the runaway behavior of their canonical counterparts. Moreover, non-minimal Kähler potentials can induce the existence of new runaway directions. These directions imply that the non-supersymmetric vacua is metastable, and the life-time of the vacuum depends on the form of the Kähler potential.

A. One-loop effective potential for non-linear sigma models.

In this appendix we calculate the Coleman-Weinberg effective potential [11] for a sigma model with general Kähler potential K and superpotential W. In [11], the computation is made for renormalizable theories, so we must recalculate it. The model we consider has N superfields Z^a, with scalar component z^a and fermionic component ψ^a. The action of the

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theory is

$$\mathcal{S} = \int d^4x \left[\int d^2\theta d^2\bar{\theta} K(Z,\bar{Z}) + \int d^2\theta W(Z) + \int d^2\bar{\theta} W(\bar{Z}) \right] . \tag{A.1}$$

Recalling that for a Kähler manifold the covariant derivative, the connection and the curvature take the form

In terms of the quantities

$$\begin{aligned}
V &= \bar{W}_{\bar{a}} K^{\bar{a}a} W_a \\
D_a W_b &= \partial_a W_b - \Gamma^c_{ab} W_c \\
\Gamma^c_{ab} &= K^{\bar{c}c}\, \partial_a\, K_{b\bar{c}} \\
D_\mu \psi^a &= \partial_\mu \psi^a - \Gamma^a_{bc} \partial_\mu z_b \psi_c \\
(R_{\bar{b}b})^{a\bar{a}} &= K^{\bar{a}c} \partial_{\bar{b}} \Gamma^a_{bc} \,,
\end{aligned} \tag{A.2}$$

and after integrating the $\theta, \bar{\theta}$ variables and the auxiliary fields, we obtain the action

$$\begin{aligned}
\mathcal{S} = \int d^4x \Big[& K_{a\bar{a}} \left(\partial_\mu z^a \partial^\mu \bar{z}^{\bar{a}} + \frac{i}{2} D_\mu \psi^a \sigma^\mu \bar{\psi}^{\bar{a}} - \frac{i}{2} \psi^a \sigma^\mu D_\mu \bar{\psi}^{\bar{a}} \right) - V(z^a, \bar{z}^{\bar{a}}) \\
& - \frac{1}{2} D_a W_b \psi^a \psi^b - \frac{1}{2} D_{\bar{a}} \bar{W}_{\bar{b}} \bar{\psi}^{\bar{a}} \bar{\psi}^{\bar{b}} + \frac{1}{4} R_{\bar{a}a\bar{b}b} \psi^a \psi^b \bar{\psi}^{\bar{a}} \bar{\psi}^{\bar{b}} \Big]
\end{aligned} \tag{A.3}$$

Being z_0^a the VEV of the scalar fields, we define small fluctuations $z^a \to z_0^a + \sqrt{\hbar}\varphi^a$ and $\psi_a \to \sqrt{\hbar}\chi_a$, and to order $\mathcal{O}(\hbar)$ we are left with

$$\begin{aligned}
\mathcal{S}^{(1)} = \int d^4x \Big[& K_{a\bar{a}}(z_0) \left(\partial_\mu \varphi^a \partial^\mu \bar{\varphi}^{\bar{a}} + \frac{i}{2} \partial_\mu \chi^a \sigma^\mu \bar{\chi}^{\bar{a}} - \frac{i}{2} \chi^a \sigma^\mu \partial_\mu \bar{\chi}^{\bar{a}} \right) - \frac{1}{2} D_a W_b(z_0) \chi^a \chi^b \\
& - \frac{1}{2} D_{\bar{a}} \bar{W}_{\bar{b}}(z_0) \bar{\chi}^{\bar{a}} \bar{\chi}^{\bar{b}} - \frac{1}{2} \left(\partial_a \partial_b V(z_0) \varphi^a \varphi^b + \partial_{\bar{a}} \partial_{\bar{b}} V(z_0) \bar{\varphi}^{\bar{a}} \bar{\varphi}^{\bar{b}} + 2 \partial_a \partial_{\bar{b}} V(z_0) \varphi^a \bar{\varphi}^{\bar{b}} \right) \Big]
\end{aligned} \tag{A.4}$$

In terms of the N scalar fields Φ^a and the N Dirac spinors Ψ^a given by

$$\Phi^a = \begin{pmatrix} \varphi^a \\ \bar{\varphi}^a \end{pmatrix} , \quad (\Phi^a)^\dagger = (\bar{\varphi}^a \quad \varphi^a) \,, \quad \Psi^a = \begin{pmatrix} (\chi^a)_\alpha \\ (\bar{\chi}^a)^{\dot{\alpha}} \end{pmatrix} , \quad \bar{\Psi}^a = -(\Psi^a)^\dagger \gamma^0 = ((\chi^a)^\alpha \quad (\bar{\chi}^a)_{\dot{\alpha}}) \,, \tag{A.5}$$

where we choose the Weyl basis for the γ-matrices

$$\gamma^\mu = \begin{pmatrix} 0_2 & \sigma^\mu \\ \bar{\sigma}^\mu & 0_2 \end{pmatrix} , \qquad \sigma^\mu = (-1, \sigma^i) \,, \qquad \bar{\sigma}^\mu = (-1, -\sigma^i) \,, \tag{A.6}$$

we can rewrite $\mathcal{S}^{(1)}$ as

$$\mathcal{S}^{(1)} = \frac{1}{2} \int d^4x \left[\partial_\mu (\Phi^a)^\dagger \mathcal{K}^B_{ab} \partial^\mu \Phi^b - (\Phi^a)^\dagger (\mathcal{M}^2_B)_{ab} \Phi^b - i \bar{\Psi}^a \mathcal{K}^F_{ab} \gamma^\mu \partial_\mu \Psi^b - \bar{\Psi}^a (M_F)_{ab} \Psi^b \right] \tag{A.7}$$

where the matrices $\mathcal{K}^B_{ab}$ and $\mathcal{K}^F_{ab}$ are defined by

$$\mathcal{K}^B_{ab} = \begin{pmatrix} K_{ba} & 0 \\ 0 & K_{ab} \end{pmatrix} \qquad \mathcal{K}^F_{ab} = \begin{pmatrix} K_{ab} \mathbf{1}_2 & 0 \\ 0 & K_{ba} \mathbf{1}_2 \end{pmatrix} = \mathcal{K}^B_{ba} \otimes \mathbf{1}_2 \tag{A.8}$$

while the mass matrices for bosons and fermions can be written as

$$\mathcal{M}_B^2 = \begin{pmatrix} D_{\bar{b}}\bar{W}_{\bar{a}}K^{\bar{a}a}D_bW_a - \bar{W}_{\bar{a}}(R_{\bar{b}b})^{a\bar{a}}W_a & \partial_{\bar{b}\bar{c}}(\bar{W}_{\bar{a}}K^{\bar{a}a})W_a \\ \bar{W}_{\bar{a}}\partial_{bc}(K^{\bar{a}a}W_a) & D_{\bar{b}}\bar{W}_{\bar{a}}K^{\bar{a}a}D_bW_a - \bar{W}_{\bar{a}}(R_{\bar{b}b})^{a\bar{a}}W_a \end{pmatrix}, \tag{A.9}$$

$$M_F = \begin{pmatrix} D_bW_a\mathbf{1}_2 & 0 \\ 0 & D_{\bar{b}}\bar{W}_{\bar{a}}\mathbf{1}_2 \end{pmatrix}. \tag{A.10}$$

Therefore, the first order correction to the effective potential reads

$$V_{\text{eff}}^{(1)} = -\log\left(\det{}^{-\frac{1}{2}}\left(\hat{\mathcal{B}}\right)\ \det{}^{\frac{1}{2}}\left(\hat{\mathcal{F}}\right)\right) \tag{A.11}$$

where we have introduced the operators

$$\hat{\mathcal{B}}_{ab} = -\mathcal{K}_{ab}^B\Box - (\mathcal{M}_B^2)_{ab}\,, \qquad \hat{\mathcal{F}}_{ab} = \gamma^0(-i\mathcal{K}_{ab}^F\ \not{\partial} - (M_F)_{ab})\,. \tag{A.12}$$

After passing to momentum space, the one-loop correction to the potential reads

$$V_{\text{eff}}^{(1)} = \frac{1}{2}Tr\int\frac{d^4p}{(2\pi)^4}\left[\log(\mathcal{K}_{ab}^Bp^2 - (\mathcal{M}_B)_{ab}^2) - \log(\gamma^0(-\mathcal{K}_{ab}^F\ \not{p} - (M_F)_{ab}))\right]. \tag{A.13}$$

Introducing the 2×2 mass matrix for fermions $\mathcal{M}_F$ through the definition

$$\mathcal{M}_F\otimes\mathbf{1}_2 = -\gamma^0M_F\,, \tag{A.14}$$

and using the properties

$$\gamma^0\mathcal{K}_{ab}^F\gamma^0 = \mathcal{K}_{ab}^{F\ \dagger} = \mathcal{K}_{ab}^B\otimes\mathbf{1}_2\,, \qquad [\mathcal{K}_{ab}^B\otimes\mathbf{1}_2, \gamma^0\gamma^\mu] = 0 \tag{A.15}$$

we can rewrite the expression for $V_{\text{eff}}^{(1)}$ as

$$V_{\text{eff}}^{(1)} = \frac{1}{2}Tr\int\frac{d^4p}{(2\pi)^4}\left[-\log(\mathcal{K}_{ab}^B) + \log(p^2\mathbf{1}_2 + \tilde{\mathcal{M}}_B^2) - \log(-\gamma^0\ \not{p} - \tilde{\mathcal{M}}_F\otimes 1_2)\right] \tag{A.16}$$

where

$$\begin{aligned} \tilde{\mathcal{M}}_B^2 &= \mathcal{K}_B^{-1/2}\mathcal{M}_B^2\mathcal{K}_B^{-1/2} \\ \tilde{\mathcal{M}}_F &= \mathcal{K}_B^{-1/2}\mathcal{M}_F\mathcal{K}_B^{-1/2}\,. \end{aligned} \tag{A.17}$$

As usual one can express the trace of the Dirac operator as the trace of a Klein-Gordon operator, i.e.

$$Tr\!\int d^4p\ \log(-\gamma^0\ \not{p} - \tilde{\mathcal{M}}_F\otimes 1_2) = 2\,Tr\!\int d^4p\ \log(p^2\mathbf{1}_2 - \tilde{\mathcal{M}}_F^2) \tag{A.18}$$

which yields the following form for the one-loop correction to the potential in Euclidean signature

$$V_{\text{eff}}^{(1)} = \frac{1}{2}Tr\int\frac{d^4p}{(2\pi)^4}\left[-\log(\mathcal{K}^B) + \log(p^2\mathbf{1}_2 + \tilde{\mathcal{M}}_B^2) - \log(p^2\mathbf{1}_2 + \tilde{\mathcal{M}}_F^2)\right] \tag{A.19}$$

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Finally, using that $d^4p = p^3\ dp\ d\Omega$, $\int d\Omega = 2\pi^2$, and

$$\int dpp^3 \log[p^2+m^2] = \frac{1}{4}(p^4-m^4)\log(p^2+m^2) + \frac{p^2m^2}{4} - \frac{p^4}{8}\,, \qquad \text{(A.20)}$$

we obtain the desired formulae after cutoff regularization

$$\begin{aligned} V_{\rm eff}^{(1)} = \frac{1}{64\pi^2}\mathrm{Tr}\Bigg(\tilde{\mathcal{M}}_B^4\left[\log\left(\frac{\tilde{\mathcal{M}}_B^2}{\Lambda^2}\right)-\frac{1}{2}\right] - \tilde{\mathcal{M}}_F^4\left[\log\left(\frac{\tilde{\mathcal{M}}_F^2}{\Lambda^2}\right)-\frac{1}{2}\right] \\ +\frac{\Lambda^2}{2}\left(\tilde{\mathcal{M}}_B^2-\tilde{\mathcal{M}}_F^2\right) - \Lambda^4\log(\mathcal{K}^B)\Bigg)\,. \end{aligned} \qquad \text{(A.21)}$$

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